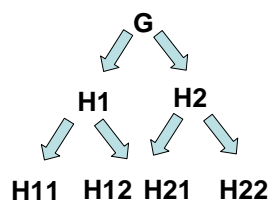


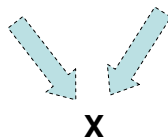
Confluence and Critical Pair Analysis in Graph Transformation Systems

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Foundations of Model Driven System
Development

What is Confluence?



Do transformations exist which lead to one common graph?

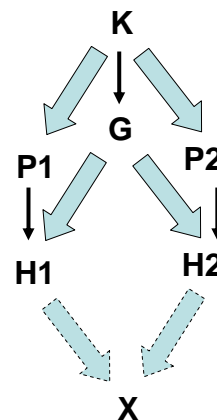


GTS is called confluent, if for all (typed) graph transformations $G \xRightarrow{*} H_1$ and $G \xRightarrow{*} H_2$ there is a (typed) graph X together with (typed) graph transformations $H_1 \xRightarrow{*} X$ and $H_2 \xRightarrow{*} X$.

- *Global determinism*: for each pair of terminating (typed) graph transformations $G \Rightarrow^* H1$ and $G \Rightarrow^* H2$ with the same source graph also the target graphs $H1$ and $H2$ are equal or isomorphic,
- $G \Rightarrow^* H$ is called *terminating* if no (typed) graph production in the GTS is applicable to H any more.
- Each confluent (typed) graph transformation system is globally deterministic.

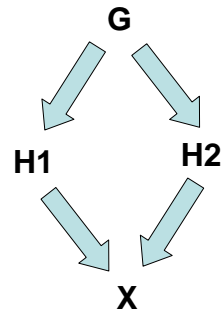
How to show confluence?

- A system is *locally confluent* if all conflicting pairs of transformations are confluent.
- A local confluent and terminating system is *confluent*.
- To check local confluence it is enough to check all *critical pairs*, i.e. minimal conflicting situations.



Local Church-Rosser Theorem

- $G \rightarrow H1$ and $G \rightarrow H2$ independent
- Then $H1 \rightarrow X$ and $H2 \rightarrow X$ exist
- *shown for labeled graphs*
- *can be rephrased for typed graphs*

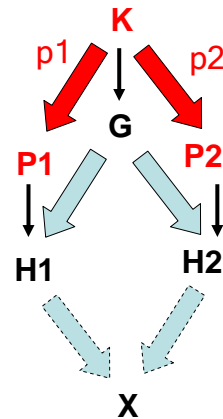


Local confluence

- Local confluence, where the given (typed) graph transformations $G \Rightarrow H1$ and $G \Rightarrow H2$ are direct (typed) graph transformations, but $H1 \Rightarrow^* X$ and $H2 \Rightarrow^* X$ are still general.
- Each terminating and locally confluent (typed) graph transformation system is already confluent.

Critical Pairs

- Two rules p_1 and p_2 with attributes over $\Sigma T(X)$ and disjoint subsets of X
- $K \rightarrow P_1$ and $K \rightarrow P_2$ is a CP candidate if non-independent, K is Σ attributed over $T(X)$
- A critical pair is a minimal CP candidate (minimal graph and maximal



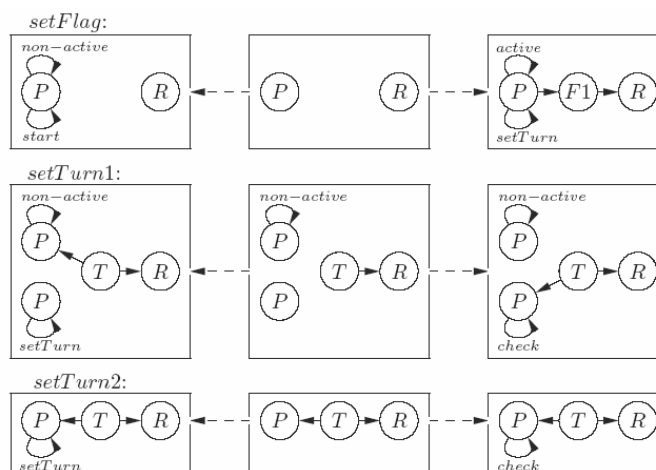
Critical pairs

- A pair $P_1 \leftarrow K \Rightarrow P_2$ of direct (typed) graph transformations is called a *critical pair*, if it is non-parallel independent and minimal in the sense that each item in K has a preimage in L_1 or L_2 (K can be considered as a suitable gluing of L_1 and L_2).
- Completeness of critical pairs:
For each pair of non-parallel independent direct graph transformations $H_1 \leftarrow G \Rightarrow H_2$, there is a critical pair $P_1 \leftarrow K \Rightarrow P_2$

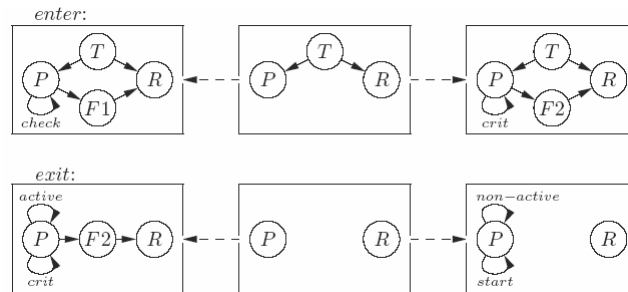
Critical pairs and local confluence

- Strict confluence: the largest subgraph N of K , which is preserved by the critical pair $P1 \leftarrow K \Rightarrow P2$, is also preserved by $P1 \Rightarrow^* K'$ and $P2 \Rightarrow^* K'$.
- Theorem:
A (typed) graph transformation system GTS is locally confluent, if all its critical pairs are strictly confluent.

Example: Mutual Exclusion



Example: Mutual Exclusion



Example

