CHARACTERIZATION OF THE SYNCHRONIZED MULTI-SINE ANALYSIS AND SYNTHESIS[1]

Annamária R. Várkonyi-Kóczy

Department of Measurement and Instrument Engineering, Technical University of Budapest, Hungary
koczy@mmt.bme.hu

ABSTRACT

Nowadays, due to advanced signal processing methods, the significance of multi-sine system identification techniques has considerably increased because they offer an excellent compromise of measurement speed and accuracy. Based on a recently suggested fast polyphase filter-bank pair synchronized multi-sine synthesis and analysis can be performed with high accuracy and low computational complexity. This solution, however, may not sufficient enough when relatively large measurement noise is also present. A simple extension providing a fading memory effect for every measuring channel has been already elaborated which can be considered as the block-oriented version of the widely used exponential averaging. Further improvements can be achieved by a simple time-varying system which can be derived as the block-oriented version of a linear averager. In this paper the development of this latter is provided and its features are examined. It is shown that for integer number of blocks the results are in complete correspondence with the ideal averager while otherwise an exponential averaging effect can be identified. If this averaging is applied to one of the channels of an N-point DFT filter-bank then after M blocks of input data the result will be an $N * M$-point DFT value.

1. INTRODUCTION

Multi-sine signals can serve as excellent perturbation signals in measurements where the contradictory requirements of speed and accuracy require a balanced solution [1]. These signals consist of harmonically related sine waves with carefully designed magnitude and phase parameters to produce composite signals with low peak factor. This latter is important because from measurement accuracy point of view it is desirable to maximize the total input signal power. The measurements to be performed provide a set of complex numbers together with their uncertainty characterization. The complex numbers give estimates of the transfer characteristic or function of the system to be identified at the frequencies of the sine waves. The concept of synchronized synthesis and analysis helps to avoid the side-effects of the input frequency and measuring channel center-frequency mismatch problem typical in the conventional discrete Fourier analysis. Theoretically both the picket-fence and the frequency leakage effects can be totally eliminated.

Recently an efficient polyphase filter-bank pair has been suggested [2] which is completely suitable for this purpose if the relative frequencies of the signal components coincide with those of the standard Discrete Fourier Transformation (DFT). The first filter bank serves as a signal synthesizer having as input the parameters of the multi-sine, while its pair is a DFT filter-bank producing the Fourier representation of the analyzed signal (see Figure 1). Since the signal generation and analysis are performed in a synchronized way and they have the very same internal structure, in practical realizations the systematic errors due to these components can be drastically reduced.

In practice, however, measurement noise is unavoidable and directly influences the uncertainty of the complete measurement. Noise reduction is possible at the prize of further measuring time. The suggestion in [3] extends the DFT filter-bank with a fading memory effect, which contributes to the noise reduction in a similar way as the exponential averaging does. This technique has been introduced also into the different versions of the fast filter-bank approach, as well [4], [5].

In this paper a time-varying extension is proposed to all the previous results. This extension results in a block-recursive linear averaging effect, which reduces the effect of noise if the measurement time increases. The operation can be characterized by the zero frequency channel output of the modified DFT filter-bank

$$y(n) = \frac{1-a(n)}{N}\,[\sum_{k=0}^{N-1} x(n-k)] + a(n)y(n-N) \quad , \qquad (1)$$

where n is the discrete time index, $x(n)$ the input signal, N the size of the DFT, and $a(n) = 1-1/q$ with $q = \mathrm{int}[n/N + 1]$. At the end of every Nth period this system will give the corresponding Fourier component calculated from the whole history of the input signal in hand. The same effect is valid for all the N channels and considerably improves the overall noise reduction capability of the filter-bank.

Section 2 summarizes the previous results, while Section 3 gives the characterization of the new extension. Finally Section 4 is devoted to simple illustrative examples.

2. SYNCHRONIZED MULTI-SINE MEASUREMENTS

The basic idea of synchronized multi-sine measurements is related to the simple fact that using digital signal processing methods synthesizer-analyzer structure pairs can be developed, and thus signal generation and analysis can be directly coupled. The simplified block diagram of this concept is given in Figure 1. Here the synthesizer is a digital signal generator which converts the $X_0,\ X_1,\ \dots,X_{N-1}$ values into a discrete sequence of N samples, periodically. These samples are converted to an analog signal and used after conditioning and possibly frequency-transpositioning to drive

[1]This work was supported by the Hungarian Fund for Scientific Research (OTKA) under contract T 017448.

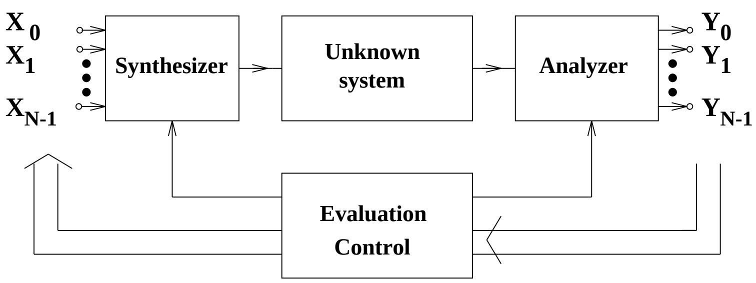


Figure 1. Block diagram of the multi-sine measuring setup

the unknown system. The output of this latter is fed to the input of the analyzer consisting of signal conditioning and demodulating circuits, and A/D converters.

The synchronized operation can be easily characterized for the DFT/FFT-based synthesizer-analyzer filter-banks. Here the generated signals consist of harmonically related sinusoids and the center frequencies of the analyzer channels are in complete correspondence with the frequencies of the input signal components. If we neglect secondary effects the whole setup is free of systematic errors inherent in usual DFT/FFT-based measurements. Additionally a self-calibration is also possible if we measure the signal of the synthesizer using the synchronized analyzer. These measurements can be used for compensations while optimizing the overall performance.

There are several alternatives to implement the synthesizer-analyzer pairs. As a result of some recent investigations a special polyphase filter-bank pair can also be considered [2], [5]. The computational complexity of these filter-banks is in direct correspondence to that of the fast transformation algorithms, but further advantages can also be mentioned. Using fast transformations typically a "block-oriented" way of thinking is applied where the transformation is performed as a single operation on given data-blocks, i.e. data-acqusition and processing are separated from each other. Using the "filter-bank" approach these two operations can be interlaced and the necessary calculations can be started immediately as the corresponding input sample arrives. This means that the overall computational load along time can be better distributed and therefore the practical performance improves. It is interesting to note that the well-known fast transformation complexity figure of $N \log_2 N$ for the proposed filter-bank will appear as $\log_2 N$ operations in every subsequent N sampling intervals. There are applications where the sliding window mode of operation is prefered. The new fast filter-bank is efficient also in these cases.

The standard signal transformations correspond to finite impulse response (FIR) filter-banks. In [4] it is shown that these filter-banks can be easily extended to have a fading memory [3] which improves the noise reduction capabilities of the transformer banks. The time-domain description of this modification can be derived directly from equation (1). If the coefficient $a(n)$ is a constant, i.e. $a(n) = a,\ 0 < a < 1$, then this algorithm will provide an exponential averaging effect on the data blocks of size N. The frequency domain interpretation of this algorithm can also be derived simply by using the z-transform of equation (1). The modified filter-bank will consist of filters with narrower passbands and smaller side-lobes and therefore the noise reduction will increase by a constant value depending on parameter a. The price to be paid for this improvement is the larger settling time.

3. TIME-VARYING BLOCK-RECURSIVE AVERAGING

The fading memory effect caused by the exponential averaging has a fixed bandwidth in every channel of the analyzer-bank and therefore can reduce noise only to a fixed rate. The idea of the time varying block-recursive averaging is simply to increase noise reduction with time, i.e. to reduce the corresponding bandwidth of the channels. The formulation of a possible algorithm can be based on equation (1). Here a simple block-averager is described, however, it is very easy to extend this formula for every DFT channel. The mth channel output $y(n,m)$ can be expressed by

$$y(n,m) = (1 - a(n))\mathrm{DFT}(n,m) + a(n)y(n-N,m) \quad , \qquad (2)$$

where

$$\mathrm{DFT}(n,m) = \frac{1}{N}\sum_{k=0}^{N-1} x(n-k)e^{-j2\pi m(N-1-k))} \qquad (3)$$

the sliding DFT of the last N input samples.

The time-varying coefficient $a(n)$ determines the properties of the averaging effect. The most obvious strategy can be to keep it constant for every block and to change it in every Nth step. The ideal linear averaging will be performed for integer number of blocks if

$$a(n) = 1 - \frac{1}{q} \quad \text{with} \quad q = \mathrm{int}[\frac{q}{N} + 1] \qquad (4)$$

$n = 0, 1, \ \ldots$. If this type of averaging is applied to the DFT channels then after M blocks of input data (2) will give the $N * M$-point DFT value. This strategy is quite reasonable if a DFT is to be performed with a maximum overall record-size of e.g. 1024 or 2048 points. For larger data size the bandwith of the DFT channels will be extremely narrow and therefore measurement errors may increase. This is typical with standard Fourier analysis since the input signal to be measured in general is not completely stationary. Changes may be followed with appropriate measurement bandwith. As a simple solution the strategy of (4) can be changed to that of the exponential averaging if $n > N_o$, i.e.

$$a(n) = 1 - \frac{1}{q} \quad \text{with} \quad q = \mathrm{int}[\frac{N_o}{N} + 1] \quad . \qquad (5)$$

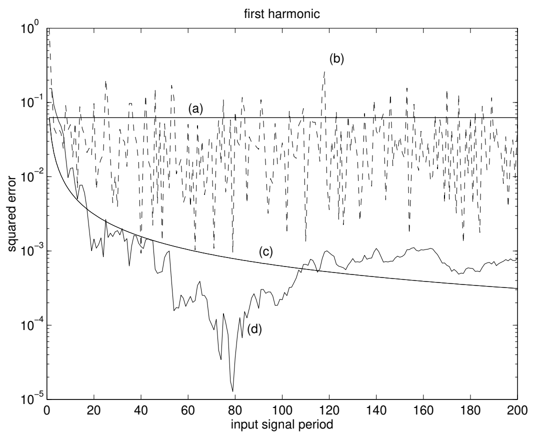


Figure 2. Squared error for the first channel in case of a single sinusoid waveform input signal, NSR=100 %, (a) ideal noise reduction rate for uncorrelated input data with block-oriented averaging, (b) simulation results for $q = 3$, (c) ideal noise reduction rate for block-oriented linear averaging, (d) simulation results

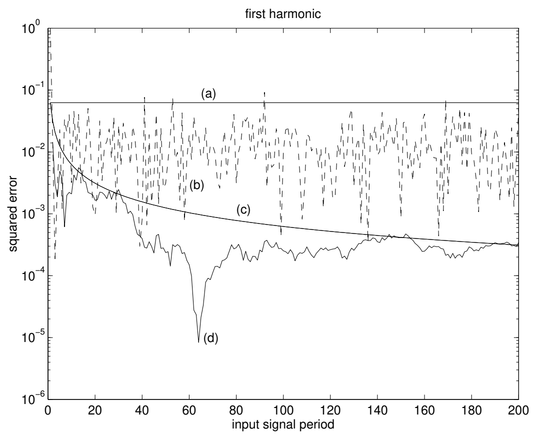


Figure 3. Squared error for the first channel in case of triangular input signal, NSR=50 %, (a) ideal noise reduction rate for uncorrelated input data with block-oriented averaging, (b) simulation results for $q = 3$, (c) ideal noise reduction rate for block-oriented linear averaging, (d) simulation results

If the non-stationary behavior can be detected somehow then some control mechanism to set the appropriate $a(n)$ can also be designed. A classical example can be the application of the Kalman filtering concept [6]. With the synchronized multi-sine setup described above, the bandwith problem of the DFT channels is smaller, since the signal to be measured has uniform line spectrum. However, all demands concerning higher measurement accuracy can be fulfilled only if the errors of the numerical calculations can be kept below some level. The system on Figure 1 offers a nice solution to some of these numerical problems, since both the synthesizer and the analyzer parts utilize the very same sine and cosine values to generate and measure, respectively. If there is a quantization error in the numerical representation of these values shifting the frequencies of the actual sine components to slightly different positions, the very same shift will appear with the center frequencies on the analyzer side.

4. ILLUSTRATIVE EXAMPLES

Figure 2 shows the simple case of a sine wave with 100 % noise. Simulations were performed with MATLAB. The additive "noise" was generated by the built-in random number generator to provide random samples between -1 and +1 [2rand(1)-1]. The frequency of the sine-wave was adjusted to the DFT channel of the first harmonic which is called "first channel". On the figure curve "a" represents the ideal noise reduction rate with time for uncorrelated input data if block-oriented exponential averaging is applied. Curve "b" gives the results of the noise reduction for the experiment performed. Curve "c" stands for the ideal noise reduction rate of the block-oriented linear averaging, while curve "d" shows the noise reduction within the experiment.

On Figure 4 the time signals are presented. Each period of the input consists of 16 samples and a 16-point DFT analysis with block-recursive linear averaging was performed. The output signal is composed as the sum of the DFT channels. Figure 3 and 5 stand for the same experiment with a triangular input and 50 % additive noise.

5. CONCLUSIONS

In this paper the general properties of the synchronized multi-sine analysis and synthesis are characterized and some additional averaging techniques are investigated. It is shown that the time-recursive DFT/FFT analyzers can be easily extended to perform block-recursive averaging to improve the noise reduction capabilities of the analyzers. The extension of this technique to other discrete transformations is straightforward.

REFERENCES

[1] K. Godfrey, *Perturbation Signals for System Identification*, Prentice-Hall International (UK) Ltd., 1993.

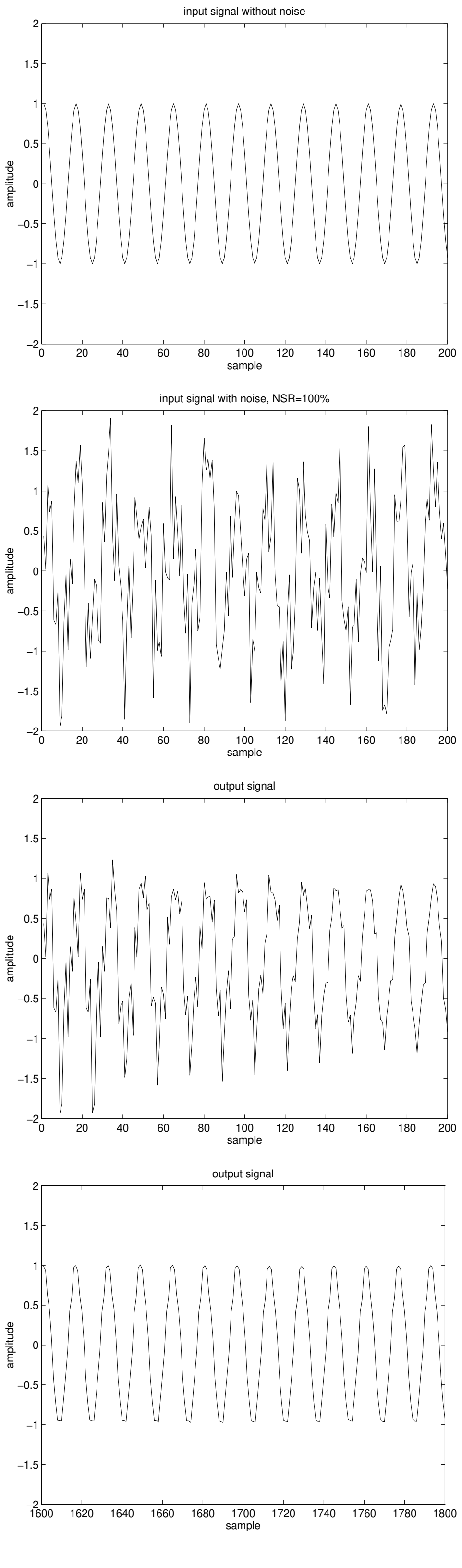
[2] A.R. Várkonyi-Kóczy, "A Recursive Fast Fourier Transformation Algorithm", *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing*, Vol. 42, No. 9, pp. 614-616, Sept. 1995.

[3] G.H. Hostetter, "Recursive Discrete Fourier Transform with Fading Memory", *Proceedings of the IEEE International Symposium on Circuits and System, ISCAS'83*, pp. 80-85, 1983.

[4] A.R. Várkonyi-Kóczy, "Efficient Polyphase DFT Filter Banks with Fading Memory", *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing*, accepted for publication, 1996.

[5] A.R. Várkonyi-Kóczy, "Multi-Sine Synthesis and Analysis via Walsh-Hadamard Transformation", *Proceedings of the IEEE International Symposium on Circuits and System, ISCAS'96*, Atlanta, Vol. 2, pp. 457-460, May 12-15, 1996.

[6] R.R. Bitmead et al., "A Kalman Filtering Approach to Short-Time Fourier Analysis", *IEEE Transactions on Acoustics, Speech and Signal Processing*, Vol. 34, pp. 1493-1501, Dec. 1986.



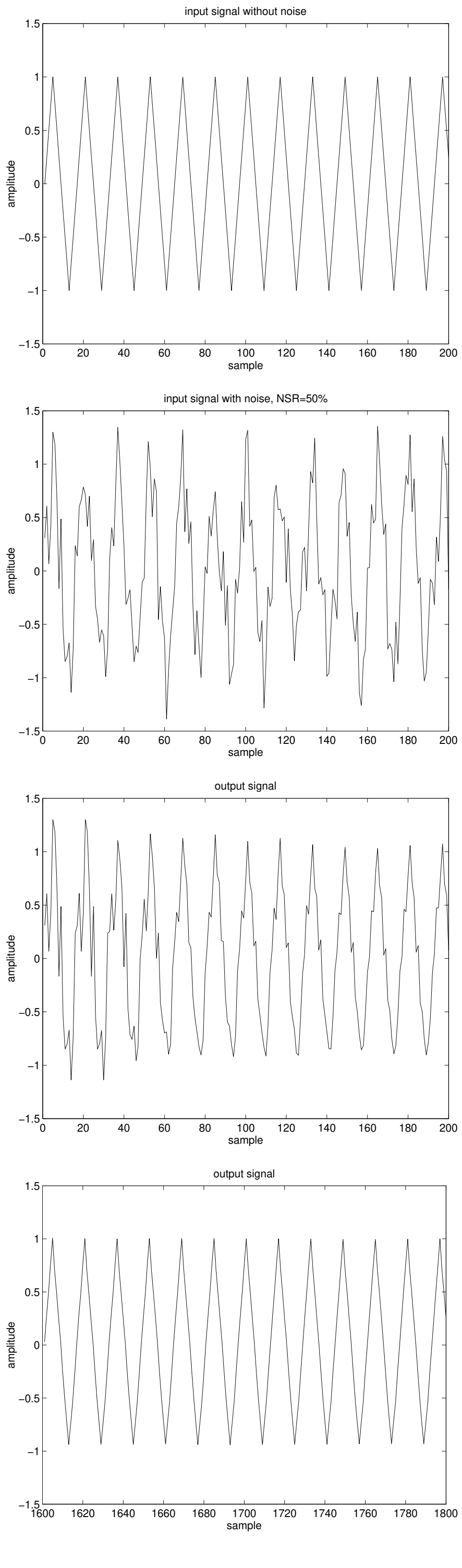


Figure 4. (a) Input signal, (b) input signal with noise (NSR=100 %), (c)-(d) output signal

Figure 5. (a) Input signal, (b) input signal with noise (NSR=50 %), (c)-(d) output signal