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Synchronized Multi-Sine Measurements via DSP Methods[1]

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Abstract— **Based on a recently suggested fast polyphase filter-bank pair a multi-sine measurement system is proposed and described. The multi-sine synthesis is performed by a multiple-input single-output filter-bank which performs the inverse discrete Fourier transformation, while as signal analyzer a single-input multiple-output filter-bank is applied. This latter calculates the discrete Fourier transform in a time-recursive manner. The computational complexity of the filter-banks corresponds to that of the fast Fourier transformation. A general characterization of the system from measurement point of view is also provided. The proposition seems to be particularly efficient when both high accuracy and speed are required.**

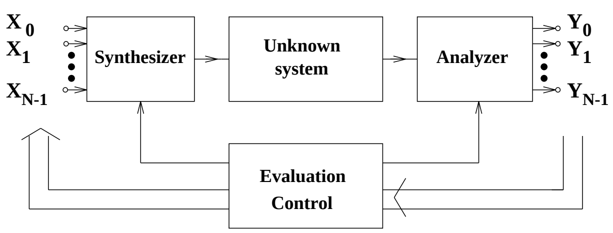

Fig. 1. Block diagram of the multi-sine measuring concept

I. Introduction

Recently the significance of multi-sine techniques in system identification has considerably increased due to some advances in the design of multi-sine perturbation signals [1] composed as a sum of harmonically related sinusoids with properly selected amplitudes and phases. It is an obvious requirement to have sinusoids with amplitudes and phases providing simultaneously maximum energy at each frequency location in hand and minimum overall crest factor for the generated signal.

In this paper the concept of a new synthesizer and analyzer pair for the practical implementation of multi-sine measurements is introduced. The novelty of this approach is signal generation and analysis via fast signal transformation pairs operated as polyphase filter-banks [2]. A typical example can be Discrete Fourier Transformation (DFT) and its inverse, but other transformations may also be considered. For an N-point DFT the signal generation is performed by an N-input single-output inverse DFT filter-bank (see Fig. 1.). The signal to be generated is coded in an input vector which has to be applied in every Nth step to provide continuous operation. The analyzer part is a synchronized one-input N-output DFT filter-bank which if applied directly for the generated signal is capable to reconstruct the generator input in every Nth step without any error.

Section II is devoted to summarize the most important features of these recently introduced fast filter-banks. Since in the majority of the practical cases these filters suffer from poor selectivity a fading memory effect [3] has been introduced [4] to improve their noise reduction capabilities. Some details of this latter are given in Section III.

For practical utilization, to produce continuous waveform, a D/A converter and a proper signal conditioning is added to the signal generator part and similarly an A/D conversion is needed at the entrance of the analyzer. In Section IV this complete setup is investigated.

The last part of the paper introduces the idea of multiple parallel A/D converters within the polyphase analyzer structure proposed in [2]. With this approach the parallel branches of the polyphase analyzer can have their own A/D converters producing the corresponding samples from the very same input signal. Since the branches operate at lower rate higher accuracy and/or overall operating speed can be achieved.

II. Fast DFT filter-banks

The DFT filter-banks are used for the time-recursive evaluation of the N-point DFT defined as

$$X_m = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}mn} \quad , \qquad (1)$$

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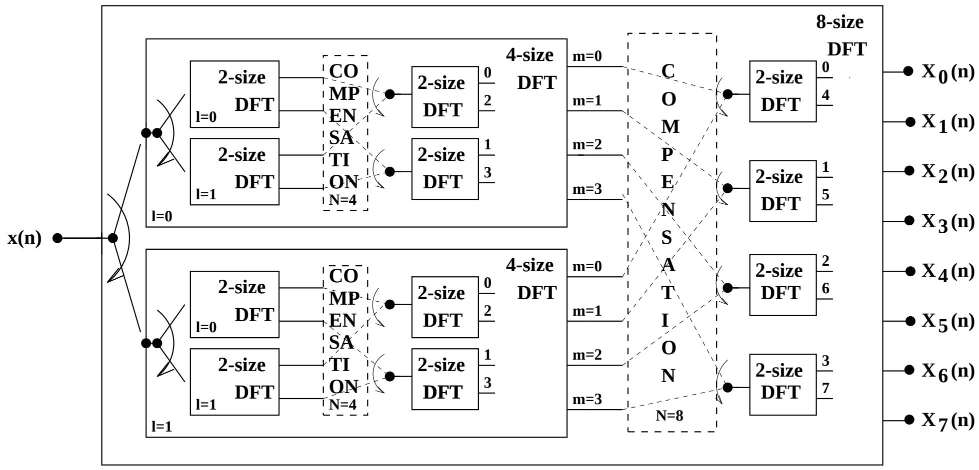

Fig. 2. Polyphase DFT analyzer for $N = 8$

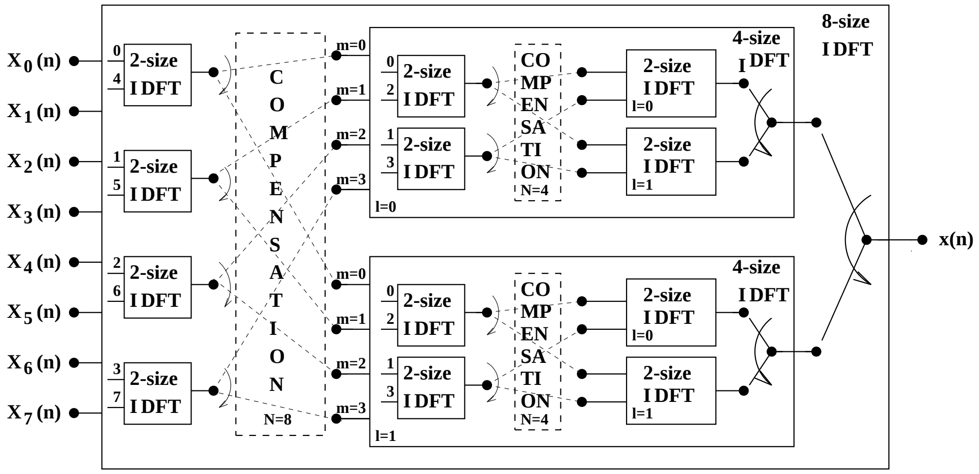

Fig. 3. Polyphase DFT synthesizer for $N = 8$

where $x(n)$ denotes the nth input sample, X_m the mth DFT component. The standard form of the recursive evaluation is simply a sliding-window technique for calculating the Fourier coefficients and/or components with respect to the last N input samples [5],[6].

Recently a new technique has been developed for the implementation of the recursive DFT based on the concepts of polyphase filtering and the fast Fourier transformation (FFT) [2]. The novelty of this approach is the decomposition of a larger size single-input multiple-output (SIMO) DFT filter-bank into a proper parallel combination of smaller ones similarly to the nondestructive zoom technique in the case of larger scale input data blocks (see e.g. [7]). The block diagram of this polyphase DFT analyzer for $N = 8$ is given on Fig. 2. This is a radix-2 case where the overall system can be decomposed into 2-point DFT blocks related by complex phase compensators between the stages. This phase compensation is unavoidable due to the delayed reference positions of the smaller size DFT blocks. The overall decomposition scheme on the input side follows the "decimation-in-time" (see e.g. [8]) while on the output side the "decimation-in-frequency" principles. The complete system corresponds to the structure of the FFT, but operates recursively, and has remarkable features in comparison with other techniques of similar complexity [9],[10]. The very same idea of decomposition can be utilized also in the case of DFT synthesis. The block diagram of the polyphase DFT synthesizer for $N = 8$ is given in Fig. 3. The synthesizer is a multiple-input single-output filter-bank where the building blocks are simple 2-point inverse DFT (IDFT) sections. The parallel input specifies the complex weights of the summed basis generating filters according to the concept of the Fourier series expansion. Since the basis generating filters produce a sequence of N samples, the very same parallel input is to be applied periodically in every Nth step if a "continuous" sequence is required.

The DFT filter-bank is well suited to the analysis of periodic signals generated by systems like in Fig. 3., since the actual frequency components of the input correspond to the discrete frequency locations of the DFT and the systematic error typically present in frequency component measurements can be completely avoided. A very simple illustration of such a measurement is given in Fig. 4. The

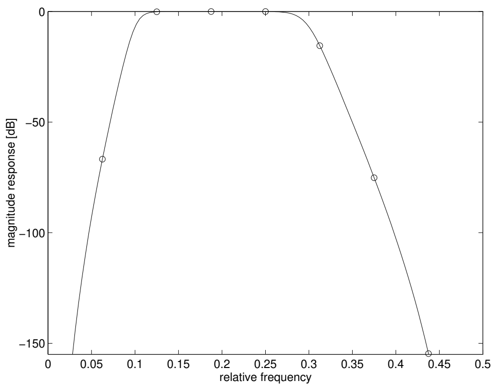

Fig. 4. Simulation results of a 10th-order bandpass Butterworth filter: (-) theoretical results, (o) simulated values.

magnitude characteristics of the individual filters within the bank, however, are not selective enough if measurement noise is also present in the signal. The noise immunity can be improved by further averaging.

III. Recursive DFT Filter Bank with Fading Memory

As it is well-known from the literature (see e.g. [5]) the averaging effect in (1) can have an interesting frequency-domain interpretation. One of the typical characterizations is the z-domain transfer function of the averager

$$\frac{1}{N}\frac{1-z^{-N}}{1-z^{-1}} \ . \tag{2}$$

This transfer function is present in every sliding DFT channel that produces the linear average of the last N (properly demodulated) input samples. A very similar characterization is valid for the DFT filter-bank

$$\frac{1}{N}\frac{1-z^{-N}}{1-z_m z^{-1}}, \quad z_m = e^{j\frac{2\pi}{N}m}, \quad m = 0,1,...,N-1, \tag{3}$$

which operates as an "averager" at the mth frequency position. The extension of the recursive DFT to a fading memory version, as it is motivated in [3],[11], can be easily solved with the application of recursive building blocks producing also certain poles for the overall transfer function. Due to implementational reasons the resonator-based approach is applied which proved to be advantageous also for higher-order blocks. If in Fig. 5.b the coefficients r_0 and r_1 are not fixed as $1/2$ we can implement the transfer function of the channels:

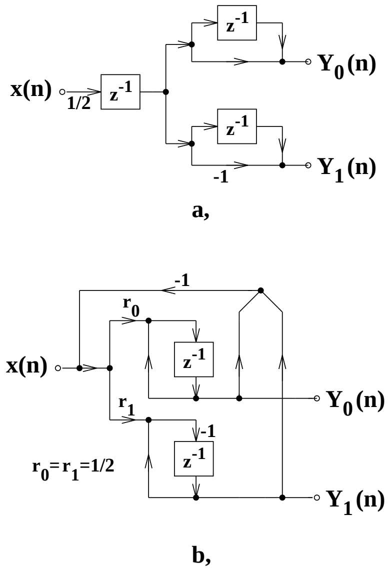

Fig. 5. 2-point DFT filter-bank versions: (a) Tree structure version, (b) Resonator-based version.

$$H_0(z) = \frac{r_0 z^{-1}(1+z^{-1})}{1+(r_0-r_1)z^{-1}+(r_0+r_1-1)z^{-2}} \ , \tag{4}$$

$$H_1(z) = \frac{-r_1 z^{-1}(1-z^{-1})}{1+(r_0-r_1)z^{-1}+(r_0+r_1-1)z^{-2}} \ . \tag{5}$$

The fading memory effect described in [3], and in [11] with an other interpretation, requires uniformly distributed poles within the unit circle. The characterization of this modified DFT system can be given if (2) is replaced by:

$$\frac{1-a}{N}\frac{1-z^{-N}}{1-az^{-N}}\frac{1}{1-z^{-1}} \ , \tag{6}$$

where the poles, which are the Nth roots of a are responsible for the fading memory effect. If we consider the behavior of the polyphase structures [12], it turns out that these poles can be generated for the radix-2 case in such a way that the first stage of the overall structure consisting of $N/2$ 2-point DFT blocks is realized using blocks of Fig. 5.b with

$$r_0 = r_1 = \frac{1-a}{2} \ . \tag{7}$$

The effects of these poles can be illustrated with the magnitude characteristics. Fig. 6. presents the magnitude behavior for different $a > 0$ values. From this Figure it turns out that in the case of synchronized sine-waves higher selectivity can be achieved, i.e. this simple modification can really serve the multi-sine perturbation approaches.

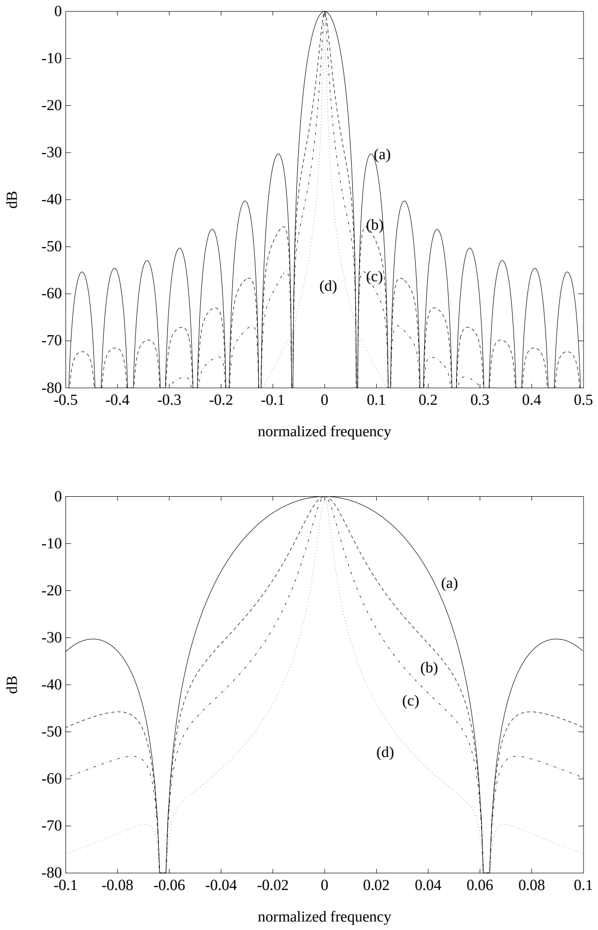

Fig. 6. Magnitude characteristics of the DFT filter-banks with different $a > 0$ parameters: (a) $a = 0$, (b) $a = 0.4$, (c) $a = 0.6$, (d) $a = 0.8$.

If not only the first stage of the radix-2 structure is replaced with the block of Fig. 5.b, but also some others, further poles can be introduced.

The proposed DFT structure together with this possibility to introduce poles can serve also as an alternative solution for signal processing problems to be solved in the transform-domain. This means the generalization of the frequency sampling concept (see e.g. [5]) related to the sliding DFT and the introduction of arbitrary pole-sets by setting the r_0, r_1 parameters properly within the last stage of the overall system. Obviously this latter can be combined with the fading memory effect of (6), as well.

IV. The multi-sine measuring system

A more detailed block diagram of the complete measuring system is given in Fig. 7. The multi-sine synthesizer and the signal analyzer operate synchronously together with the D/A and A/D converters. If frequency transposition circuits are also applied then a synchronized "carrier-band" analysis, i.e. "zoom" analysis is also possible. Concerning errors within the system if we neglect the quantization errors of the digital signal processing parts then only the side-effects of the D/A and A/D conversions and the frequency transpositions are to be considered. Fortunately frequency mismatch problems can be completely avoided since frequency transpositions can share common frequency reference. On the other hand the magnitude and phase errors of these circuits can be "measured" by the system itself, since the output of the signal generator can be directly analyzed, and the calculated difference of the multi-sine signal parameters can directly serve for correction. If the D/A and A/D converters share common voltage reference then relative to this reference the system is capable to calibrate itself automatically.

V. Multiple A/D converters within the polyphase DFT analyzer

Nowadays the accuracy of measurement systems can be considerably improved by the direct utilization of signal processing techniques. The available DSP processors can provide also good speed performance. Speed and accuracy, however, are contradictory requirements in measurements. As far as A/D conversion is concerned flash converters can operate at very high speed, but their resolution is not acceptable. Sigma-delta A/D converters provide excellent resolution and accuracy, but at the prize of low conversion rate (see e.g. [13]). For high resolution (>16 bits) measurements a possible alternative can be the application of parallel sigma-delta converters at the input of the polyphase DFT analyzer (see Fig. 2.) since its internal processing elements operate at a much lower rate. The delay caused by the converters can be easily compensated either in the phase of the generated signal or as a correction during self-calibration. If the signal processing part of the system would limit the speed of operation then the proposition in [14] may help, where signal generation and analysis is performed via the computationally extremely efficient Walsh-Hadamard transformation.

VI. Conclusions

In this paper a multi-sine measurement system structure was proposed based on fast DFT filter-banks. Due to the synchronized operation of the synthesizer and analyzer the systematic error of the DFT filter-banks can be completely eliminated. The unknown system to be measured is supposed to be continuous, therefore a D/A converter and a proper signal conditioning is added to the digital signal generation part and similarly signal conditioning and A/D conversion is needed on the analyzer side. For zoom-type analysis synchronized frequency transposition circuits are also required which for lower frequency applications can be implemented digitally, as well. The errors

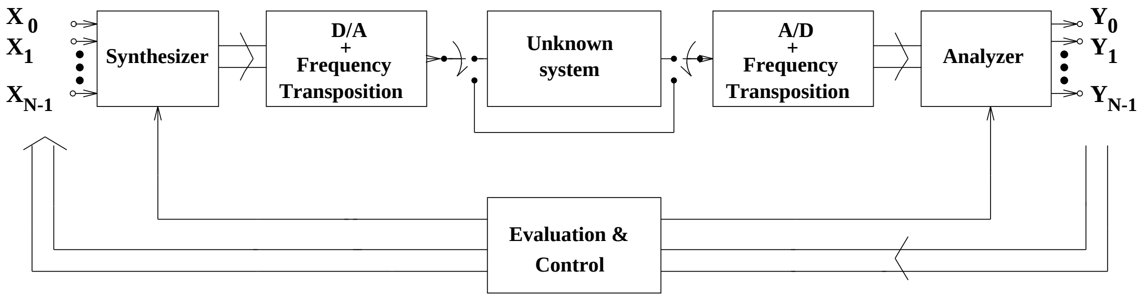

Fig. 7. Block diagram of the multi-sine measuring system

introduced by these additional circuits may be corrected by the results of self-calibration.

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