MULTI-SINE SYNTHESIS AND ANALYSIS VIA WALSH-HADAMARD TRANSFORMATION

Annamária R. Várkonyi-Kóczy

Department of Measurement and Instrument Engineering, Technical University of Budapest, Hungary
koczy@mmt.bme.hu

ABSTRACT

In this paper, based on a special filter-bank pair, a new multi-sine synthesizer and analyzer is presented which can be efficiently utilized in solving system identification problems. These filter-banks are fast, indirect implementations of the inverse discrete (IDFT) and discrete Fourier Transformation (DFT) algorithms providing low computational complexity and high accuracy. The proposed structures are based on the proper combination of polyphase filtering and the Walsh-Hadamard (WHT) Transformations. The inherent parallelism of these structures enables very high speed in practical implementations and the use of more parallel A/D, D/A converters.

1. INTRODUCTION

Although the classical solutions in digital signal processing offer very well established methods for the frequency-domain signal representation – like the discrete Fourier Transformation (DFT) and its fast algorithm the fast Fourier Transformation (FFT) [1,2] – we have to face serious limits due to the contradictory requirements of magnitude and frequency resolution. To reduce these problems recently the application of multi-sine perturbation signals came into focus [9].

Another disadvantageous aspect is that the widely used FFT techniques are operated block-orientedly, i.e. process given blocks of data, so do not directly support real-time signal-processing which also has become an important claim in this field.

Using the classical recursive DFT methods [1] the real-time processing can be solved with a higher computational burden relative to the FFT. Later the classical version based on the Lagrange structure has been replaced by an observer structure [3,4], however, computational complexity remained the same.

Recently a fast implementation of the recursive DFT has been developed [5] which combines the idea of polyphase filtering [6] and the FFT and is based on the decomposition of a larger size single input multiple output (SIMO) DFT filter-bank into proper parallel combination of smaller ones. Its computational complexity is in direct correspondence with the FFT and besides, its polyphase nature provides additional advantages in parallelization.

Using this structure a fast adaptive Fourier analyzer [7] can be derived and by applying recursive building blocks within the structure a recursive DFT with fading memory can also be implemented [8]. These latter have real importance in such system identification problems where periodic, multi-frequency perturbation signals are applied.

In this paper a dedicated structure pair is presented for the efficient solution of the above described problems. In Section II the application of synchronized signal synthesis and analysis is proposed while in section III the fast recursive synthesis and analysis structure based on the Walsh-Hadamard Transformation will be detailed.

2. SYNCHRONIZED SYNTHESIS AND ANALYSIS

In multi-sine measurements the perturbation signal of the system to be identified is a multi-sine signal and the amplitudes and phases of the harmonic components of the response are to be measured. The accuracy of the identification depends on the accuracy of the determination of the input/output ratio of the components. If we synchronize the synthesis and analysis of the harmonic components then the systematic error of the DFT/FFT methods can be eliminated. The synchronization can be solved through the use of a joint pair of signal synthesizer and analyzer. Fig. 1. shows the block diagram of the multi-sine measuring procedure.

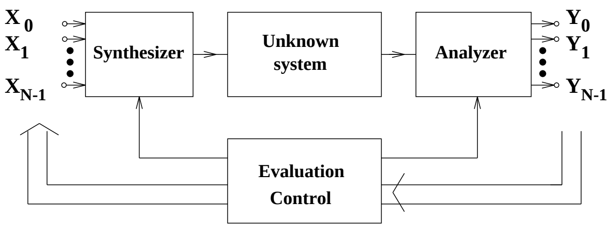

Figure 1. Block diagram of the multi-sine measuring setup

The synthesizer and analyzer units contain the signal conditioning circuits and the D/A and A/D converters, respectively. The synthesizer generates a multi-sine input signal for the system with components in given frequency-positions with maximized quasi-uniform amplitudes. This can be optimized through the appropriate setting of the phase positions [9]. The analyzer is the inverse of the synthesizer, its channels are "tuned" to the components of the multi-sine signal. The transfer function is determined by the ratio of the complex output/input values. This is evaluated by the common control unit.

[1]This work was supported by the Hungarian Fund for Scientific Research (OTKA) under contract 4-100.

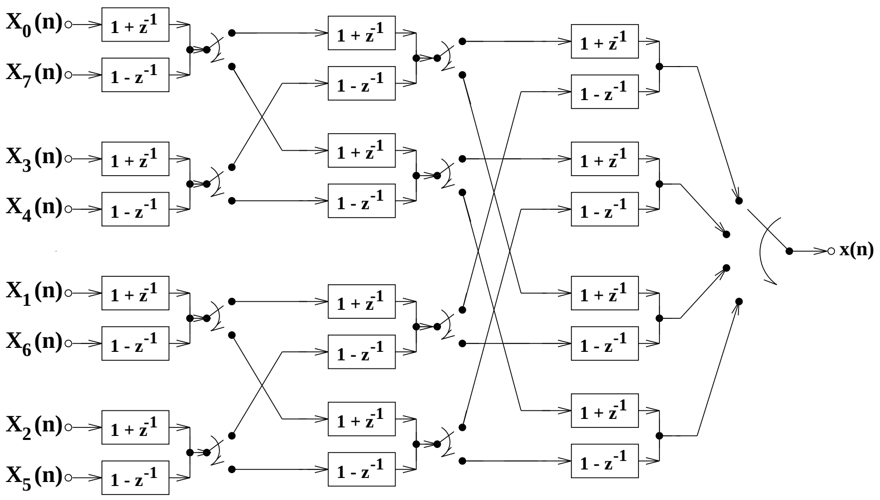

Figure 2. Block diagram of the Walsh-Hadamard synthesizer for $N = 8$

3. SYNTHESIS AND ANALYSIS VIA WALSH-HADAMARD TRANSFORMATION

The block diagram of the proposed synthesizer is given in Fig. 2. It is a multiple input single output (MISO) system which can generate arbitrary periodic waveforms. It operates like a parallel to serial converter, i.e. it has a nonzero parallel input in every Nth step and generates a sequence of N samples corresponding to the actual input. If this input is repeated in every Nth step the output will be a periodic waveform. The overall structure implements a complete weighted set of Walsh-Hadamard basis sequences in an efficient form with a complexity corresponding to that of the fast algorithms [2]. It is important to note that the input signal is the Walsh-Hadamard representation of the sequence to be generated with an accuracy depending only on the accuracy of the weights, since within the structure only additions and subtractions are to be performed. The output is obtained via a demultiplexer which can be applied to a D/A converter. The applicability of more D/A converters and an analog demultiplexer is under investigation.

As an example Fig. 3. shows a single sinusoid waveform generated using the above method. The input values of the Walsh-Hadamard synthesizer can be easily calculated as the Walsh-Hadamard transform of the time-sequence to be generated. Let's denote the vector of this sequence by $\underline{x} = [x(0), x(1), ..., x(N-1)]^T$, and the $N*N$ transformation matrix by $\underline{\underline{W}}$. The N vector to be applied in every Nth step at the input of the synthesizer is given by $\underline{X} = \underline{\underline{W}}\,\underline{x}$.

The block diagram of the analyzer structure is given in Fig. 4. It is in complete correspondence with the synthesizer and can be considered as a serial to parallel converter system maximally decimated at its output if necessary. The different channels calculate the Walsh-Hadamard coefficients corresponding to the last N input samples. The complexity remains the same as for the generation. The application of parallel A/D converters at the input, similarly to the parallel D/As, is under investigation.

If the output of the synthesizer is connected directly to the input of the analyzer, then after N steps its output values will equal the Walsh-Hadamard transform components,

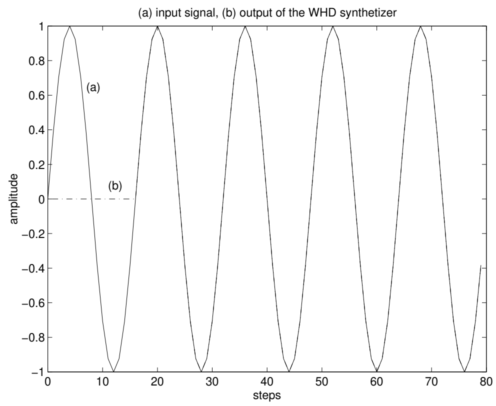

Figure 3. Single sinusoid waveform generated using WHT-synthesizer:(a) desired signal, (b) output signal.

i.e. the components of $\underline{X}$. If there is a system to be identified in between, then we can characterize the unknown system by the corresponding channel inputs $\underline{X}$ and outputs $\underline{Y}$ of the synthesizer and the analyzer, respectively. The widely used frequency domain characterization of the system, however, requires some additional computations. On the input side the vector of the complex Fourier components $\underline{X}_F$ can be calculated as

$$\underline{X}_F = \underline{\underline{F}}\,\underline{x} = \underline{\underline{F}}\,\underline{\underline{W}}^{-1}\underline{X} = \underline{\underline{V}}\,\underline{X} \ , \qquad (1)$$

where $\underline{\underline{F}}$ stands for the $N*N$ discrete Fourier Transformation (DFT) matrix, while at the output of the analyzer

$$\underline{Y}_F = \underline{\underline{F}}\,\underline{y} = \underline{\underline{F}}\,\underline{\underline{W}}^{-1}\underline{Y} = \underline{\underline{V}}\,\underline{Y} \ . \qquad (2)$$

The stationary behavior of the system to be identified can be characterized by the transfer values derived as the ratio of the corresponding components of $\underline{Y}_F$ and $\underline{X}_F$ as

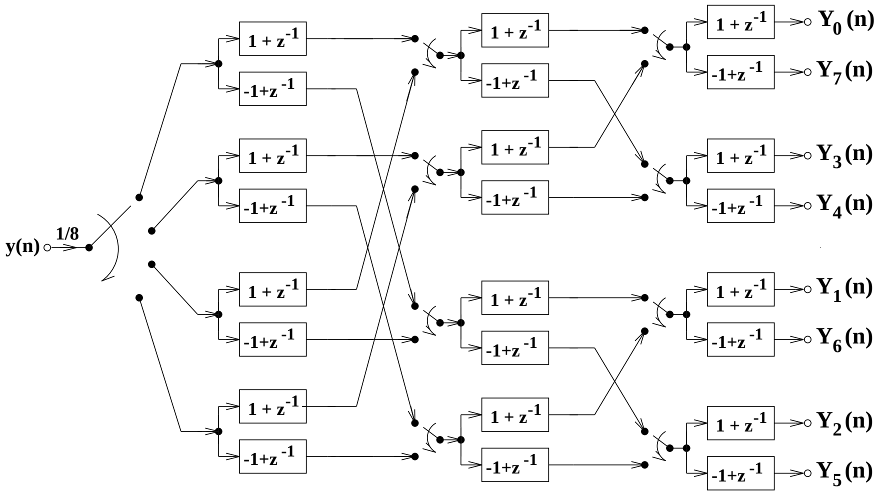

Figure 4. Block diagram of the Walsh-Hadamard analyzer for $N = 8$

the transients of the overall system die out. It is important to note that the results will be available at the end of a complete sequence of N samples, i.e. in every Nth step.

Based on equation (1) the practical measurements start with the specification of the proper multi-sine signal. This is performed by setting the proper initial magnitude and phase via the components of $\underline{X}_F$ (See [9]). The next step is the calculation of the vector $\underline{X} = \underline{\underline{V}}^{-1}\underline{X}_F$, which is directly used in the signal generation (see Fig. 2.). Finally the output of the analyzer (see Fig. 4.) should be introduced into equation (2) to get the corresponding sine-wave parameters.

The measurement setup described above is a finite impulse response (FIR) filter-bank which performs a sliding-window mode of operation, and therefore its outputs characterize always the last block of N samples, i.e. each channel of the bank can be considered as a "moving-average" filter. If we consider the effect of the sliding window also for $\underline{X}_F$ then we can extend the method to get a new measurement in every step.

In practical measurements the presence of noise is unavoidable. If we model the noise effects as an additive white noise input to the analyzer having variance σ^2, then this variance will be reduced to σ^2/N in every channel. Further improvement can be achieved if we introduce the fading memory effect described in [11] and [12]. The characterization of this effect can be given as

$$\frac{1-a}{N}\,\frac{1-z^{-N}}{1-az^{-N}}\,\frac{1}{1-z^{-1}}\ , \tag{3}$$

where the poles located at the positions determined by the Nth roots of $0 < a < 1$ are responsible for further noise reduction. The effects of these poles can be illustrated with the corresponding magnitude characteristics. (See Fig. 5. for different a values). The extension of the Walsh-Hadamard transformer to such a fading memory version can be solved with the application of simple second-order recursive blocks. If the first stage of the structure on Fig. 4. consisting of $N/2$ $2*2$ Walsh-Hadamard transformers is realized using e.g. the second-order blocks of Fig. 6. with

$$r_0 = r_1 = \frac{1-a}{2} \tag{4}$$

then the overall structure will produce the modified performance described by (3). The prize to be paid for this improvement in noise reduction is the increase of the measurement time, since the poles introduced will cause longer transients. As a simple example Fig. 7. shows the simulated measurement results of a 5th-order Butterworth low-pass filter. After 10 complete periods of the multi-sine input the measurement results show very good coincidence with the calculated "theoretical" values.

4. CONCLUSIONS

In this paper a new setup has been proposed for system identification purposes using multi-sine perturbation signals. Both the synthesizer and analyzer part of the system is implemented using the Walsh-Hadamard signal representation. Since the proposed structure is highly parallel and computationally efficient it can be recommended for real-time applications. Additional advantages:

1. The sliding-window character of the structures makes possible the implementation of continuous measurements: the analyzer outputs are not necessarily to be decimated.
2. Due to its polyphase nature it is feasible to solve polyphase A/D, D/A conversions applying more converters and thus higher sampling frequency can be reached with the same resolution. (The details are under investigation).
3. The common control unit helps the calibration and the optimization of the phase positions.

REFERENCES

[1] L.R. Rabiner, B. Gold, *Theory and Application of Digital Signal Processing*, Prentice-Hall, Englewood Cliffs, NJ, 1975.

[2] R.E. Blahut, *Fast Algorithms for Digital Signal Processing*, Addison-Wesley Publishing Company, Inc., 1985.

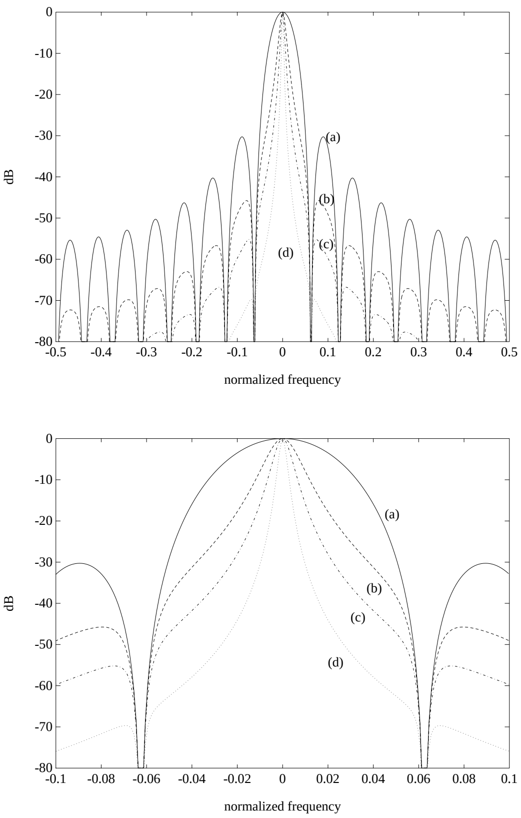

Figure 5. Magnitude characteristics of the filter-banks with different $a > 0$ parameters: (a) $a = 0$, (b) $a = 0.4$, (c) $a = 0.6$, (d) $a = 0.8$.

[3] G.H. Hostetter, "Recursive Discrete Fourier Transformation", *IEEE Transactions on Acoustics, Speech, and Signal Processing*, Vol. ASSP-28, No. 2, pp. 184-190, Apr. 1980.

[4] G. Péceli, "A Common Structure for Recursive Discrete Transforms", *IEEE Transactions on Circuits and Systems*, Vol. CAS-33, pp. 1035-1036, Oct. 1986.

[5] A.R. Várkonyi-Kóczy, "A Recursive Fast Fourier Transformation Algorithm", *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing*, Vol. 42, No. 9, pp. 614-616, Sept. 1995.

[6] R.E. Crochiere, L.R. Rabiner, *Multirate Digital Signal Processing*. Prentice-Hall, Englewood Cliffs, NJ, 1983.

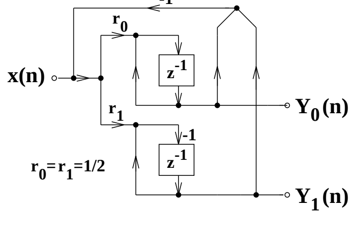

Figure 6. Second-order recursive filter block

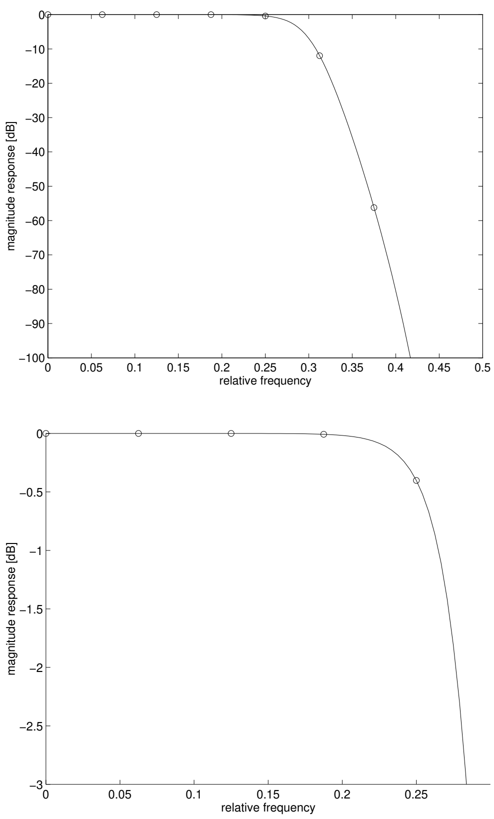

Figure 7. Simulation results of a 5th-order Butterworth low-pass filter: (-) theoretical results, (o) simulated values.

[7] A.R. Várkonyi-Kóczy, "A Composite Observer Structure for Adaptive Fourier Analysis", *Proceedings of the 5^{th} IFAC Symposium on Adaptive Systems in Control and Signal Processing, ACASP'95*, Budapest, Hungary, pp. 171-176, June 1995.

[8] A.R. Várkonyi-Kóczy, "Parallel Implementation of the Recursive Discrete Fourier Transformation", *Proceedings of the European Conference on Circuit Theory and Design, ECCTD'95*, Istambul, Turkey, pp. 999-1002, Aug. 1995.

[9] K. Godfrey, *Perturbation Signals for System Identification*, Prentice-Hall International (UK) Ltd., 1993.

[10] Y. Tadokoro, T. Higuchi, "Discrete Fourier Transform Computation Via the Walsh Transform", *IEEE Transactions on Acoustics, Speech and Signal Processing*, Vol. ASSP-26, pp. 236-240, June 1978.

[11] G.H. Hostetter, "Recursive Discrete Fourier Transform with Fading Memory", *Proceedings of the IEEE International Symposium on Circuits and System, ISCAS'83*, pp. 80-85, 1983.

[12] R.R. Bitmead, A.C. Tsoi, P.J. Parker, "A Kalman Filtering Approach to Short-Time Fourier Analysis", *IEEE Transactions on Acoustics, Speech, Signal Processing*, Vol. ASSP-34, No. 6, pp. 1493-1501, Dec. 1986.