State Dependant Anytime Control Methodology for Prototypical Aeroelastic Wing Section with Structural Non-linearity

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***Abstract* - Nowadays in solving control problems the processing is performed typically by model-based computer systems, which contain a representation of our knowledge about the nature and the actual circumstances of the problem in hand. If the nature and/or the actual circumstances change the corresponding model should also be changed. Anytime techniques are very flexible in this respect and can advantageously be used when the operation should be performed under changing circumstances. In this paper, a nonlinear state dependant control methodology is proposed for anytime use and as an example is applied to globally stabilize a given prototypical aeroelastic wing section via one control surface.**

***Index Terms*—aeroelastic wing section, nonlinear control, anytime systems, multiple models, parallel distributed compensation.**

I. INTRODUCTION

Nowadays practical solutions of engineering problems involve model-integrated computing. This integration means that the available knowledge is represented in a proper form and acts as an active component of the procedure (computer program) to be executed during the operation of the diagnostics, measuring, control, etc. devices. Thus, model based approaches offer a very challenging way to integrate a priori knowledge into the procedure.

In case of linear problems very well established methods are available and they are successfully combined with adaptive techniques to provide optimum performance. However, while several authors have investigated the effectiveness of linear control strategies for nonlinear systems, experimental evidence has shown that these control methods may not be reliable when the non-linear effects pre-dominate.

In the field of aeroelastic systems e.g. in case of large amplitude limit cycle oscillation behavior the linear control methodologies [1] do not stabilize aeroelastic systems consistently. [2] and [1] proposed non-linear feedback control methodologies for a class of non-linear structural effects of the wing section [3]. On the other hand papers [2,4] develop a controller, capable of ensuring local asymptotic stability, via partial feedback linearization and a state-dependent controller has been derived via analytic solution of the state-dependent Riccati equations of prototypical wing section in [5]. It has been shown that by applying two control surfaces global stabilization can be achieved.

When controlling a nonlinear system not only a model building can be of problem. In computer-based monitoring and control systems the operations should be performed under prescribed response time conditions. It is an obvious requirement to provide enough computational power but the achievable processing speed is highly influenced by the precedence, timing, and data access conditions of the processing itself. It seems to be unavoidable even in the case of extremely careful design to get into situations where the shortage of necessary data and/or processing time becomes serious. Such situations may result in a critical breakdown of the monitoring and/or control systems [6]. The concept of "anytime" processing tries to handle the case of too many abrupt changes and their consequences in larger scale embedded systems [7]. The idea is that if there is a temporal shortage of computational power and/or there is a loss of some data the actual operations should be continued to maintain the overall performance "at lower price", i.e., information processing based on algorithms and/or models of simpler complexity should provide outputs of acceptable quality to continue the operation of the complete system. The accuracy of the processing will be temporarily lower but possibly still enough to produce data for qualitative evaluations and supporting decisions. Consequently "anytime" processing provides short response time and is very flexible with respect to the available input information and computational power.

In this paper authors show that the nonlinear state dependant control methodology based on the Tensor Product (TP) transformation [8] and Parallel Distributed Compensation (PDC) [9] is capable to stabilize systems with structural non-linearity while the model is operated in an anytime mode flexibly coping with the temporal time/resource conditions. The method is applied to a prototypical wing section, which incorporates the essential and well-characterized structural non-linearities that yield limit cycle oscillation at low speeds.

II. ANYTIME PROCESSING

Today there is an increasing number of applications where the computing must be carried out on-line, with a guaranteed response time and limited resources. Moreover, the available time and resources are not only limited but can also change during the operation of the system.

Good examples are the modern computer-based diagnostics, monitoring, and control systems, which are able to supervise complex industrial processes and determine appropriate actions in case of failures or deviation from the optimal operational mode. In these systems the model of the supervised system is used and the evaluation of the system model must be carried out on-line, thus the model must not only be correct, but also treatable by the limited resources during limited time. Moreover, if some abnormality occurs in the system's behavior it may cause the reallocation of a part of the finite resources from the evaluation of the system model to another task. Also in case of an alarm signal, lower response time may be needed.

In these cases, the so-called anytime algorithms and systems [10] can be used advantageously, which are able to provide guaranteed response time and are flexible in respect to the available input data, time, and computational power. This flexibility makes able these systems to work in changing circumstances without critical breakdowns in the performance. Naturally, while the information processing can be maintained, the complexity must be reduced, thus the results of the computing become less accurate [11].

The models/algorithms/computing methods, which are suitable for anytime usage have to fulfill the requirements of low complexity, changeable, guaranteed response time/computational need, accuracy, and known error.

Iterative algorithms are popular tools in anytime systems, because their complexity can be easily and flexibly changed. These algorithms always give some – not accurate - result, and more and more accurate results can be obtained, if the calculations are continued. A further advantageous aspect of iterative algorithms is that we don't have to know the time/resource-need of a certain configuration in advance, the calculations can be continued until the results are needed. Then by simply stopping the calculations, always in the given conditions achievable most accurate results are obtained.

Unfortunately, the usability of iterative algorithms is limited. While there is a wide range of problems, which can be solved by iterative algorithms, adequate evaluation methods can not always be found.

Besides the iterative algorithms, in a more general frame, a wide-range of other types of computing methods/algorithms can be applied in anytime systems. This frame means that a modular architecture is used [12]. The system is composed of modules each of them offering several implementations for a given task. These units (implementations within a given module) have uniform interface (same set of inputs, outputs, and solve the same problem) but can be characterized by different attribute-values, i.e. differ in their computational need and accuracy. At a given time, in the knowledge of the temporal conditions (tasks to complete, achievable time/resources, needed accuracy, etc.) an expert system can choose the adequate configuration, i.e. the units from the modules, which will be used. This means the optimization of the whole system instead of individual modules.

Anytime processing may have great advantages in control and related fields. Besides the possibility of coping with the temporarily available resources and data it offers a way for supporting 'early' decision-making concerning the necessary control actions if e.g. faults/problems occur in the supervised system. With this not only the expenses of the maintenance can be reduced but in some cases also serious alarm situations can be prevented.

III. THE LMI AND PDC BASED CONTROLLER DESIGN METHODOLOGY

The recently proposed very powerful numerical methods (and associated theory) for convex optimization involving Linear Matrix Inequalities (LMI) help us with the analysis and the design issues of dynamic systems models in acceptable computational time [13]. One direction of these analysis and design methods is based on LMI's and parallel distributed compensation (PDC) techniques [9], and functions with the multiple-model form. In this paper we utilize the TP transformation and a PDC controller design technique to derive viable control methodologies for anytime environment. The key idea of the proposed design methodology is that the TP transformation is utilized to represent the model in multiple-model form with specific characteristics, whereupon PDC controller design techniques can immediately be executed. The detailed description of the TP transformation and PDC based designs can be found in [8,9].

The state space description of a dynamic model can be formulated as

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}(\mathbf{p}(t))\mathbf{x}(t) + \mathbf{B}(\mathbf{p}(t))\mathbf{u}(t)\\ \mathbf{y}(t) &= \mathbf{C}(\mathbf{p}(t))\mathbf{x}(t) + \mathbf{D}(\mathbf{p}(t))\mathbf{u}(t)\end{aligned} \tag{1}$$

where

$$\mathbf{S}(\mathbf{p}(t)) = \begin{pmatrix}\mathbf{A}(\mathbf{p}(t)) & \mathbf{B}(\mathbf{p}(t))\\ \mathbf{C}(\mathbf{p}(t)) & \mathbf{D}(\mathbf{p}(t))\end{pmatrix}$$

is the nonlinear system matrix, element of a real valued tensor vector space, vectors $\mathbf{x}(t)$, $\mathbf{u}(t)$, and $\mathbf{y}(t)$ are the state, input, and output vectors, respectively, further $\mathbf{p}(t)$ is time varying and is bounded by a real valued N-dimensional space Ω.

A. Multiple-model

From (1) a multiple model form, utilized in this paper, can be defined

$$\begin{pmatrix}\dot{\mathbf{x}}\\ \mathbf{y}\end{pmatrix} = \left(\sum_{r=1}^{R} w_r(\mathbf{p})\mathbf{S}_r\right)\begin{pmatrix}\mathbf{x}\\ \mathbf{u}\end{pmatrix} \tag{2}$$

where basis functions $w_r(\mathbf{p})$ fulfill:

$$\forall r, \mathbf{p} : w_r(\mathbf{p}) \in [0,1] \quad \text{and} \quad \forall \mathbf{p} : \sum_{r=1}^{R} w_r(\mathbf{p}) = 1. \tag{3}$$

This defines a fixed polytope, where the system varies in: $\mathbf{S}(\mathbf{p}) \in \{\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_R\}$. Matrices $\mathbf{S}_r$ are termed vertex systems. Polytope (3) defines the convex hull of the vertex

systems.

In many cases he basis functions $w_r(\mathbf{p})$ are decomposed to dimensions, which leads to a higher structure of (2). Having the decomposed basis multiple-model, (2) can be written in the form of

$$\begin{pmatrix}\dot{\mathbf{x}}\\ \mathbf{y}\end{pmatrix}=\left(\sum_{i_1=1}^{I_1}\sum_{i_2=1}^{I_2}\cdots\sum_{i_N=1}^{I_N}\prod_{n=1}^{N}w_{n,i_n}(p_n)S_{i_1,i_2,\ldots,i_n}\right)\begin{pmatrix}\mathbf{x}\\ \mathbf{u}\end{pmatrix},$$

which can be expressed in a more compressed tensor-product form

$$\begin{pmatrix}\dot{\mathbf{x}}\\ \mathbf{y}\end{pmatrix}=\left(S\overset{N}{\underset{n=1}{\otimes}}\mathbf{w}_n(p_n)\right)\begin{pmatrix}\mathbf{x}\\ \mathbf{u}\end{pmatrix}. \tag{4}$$

(For tensor operations see [14].) Here, the row vector $\mathbf{w}_n(p_n)$ contains the basis functions $w_{n,in}(p_n)$, the N+2-dimensional coefficient tensor S is constructed from the vertex system matrices $\mathbf{S}_{i1,i2,\ldots,iN}$. The first N dimensions of S are assigned to the dimensions of $\mathbf{p}$.

B. TP Transformation to multiple model

The TP transformation has various options. The transformation can generate e.g. "minimal", "convex", and "close-to-localized" basis. The "close-to-localized" basis means that the vertex models involved in the multiple-model form are the linear models of the given dynamic model $\mathbf{S}(\mathbf{p})$ over certain operation points, namely, the vertex systems are included in the dynamic model at certain points $\mathbf{p}$ or they are as close to the given model as possible. In order to be as close to the real system as possible, let us select the "close-to-localized" basis. Vectors $\mathbf{w}_n(p_n)$ and tensor S are defined at (4). Work [15] shows that the multiple-model (4) is no-where dense in the modeling space if the number of basis functions is bounded, which is always the case in numerical implementations, thus the transformed multiple-model is only an approximation of the original system. In general cases

$$\dot{\mathbf{x}}\underset{\varepsilon}{\approx}S\overset{N}{\underset{n=1}{\otimes}}\mathbf{w}_n(p_n)\begin{pmatrix}\mathbf{x}\\ \mathbf{u}\end{pmatrix}, \tag{5}$$

where ε denotes the transformation error. It is zero if the given model can be transformed exactly to multiple-model form. If exact representation does not exist then we should employ as many basis functions as possible to ensure small ε. The TP transformation defines the relation between ε and the number of basis functions, which helps us with optimizing the number of basis functions, subject to an acceptable error.

C. PDC Controller design

The PDC design techniques determine one feedback to each vertex model. Stability is ensured by fulfilling a stability criterion, which is expressed in terms of matrix algebra or Linear Matrix Inequalities. The control performance depends on the selected criteria. A large collection of such theorems is presented in [9]. Under the framework of vertex feedback systems, one can define the control value as:

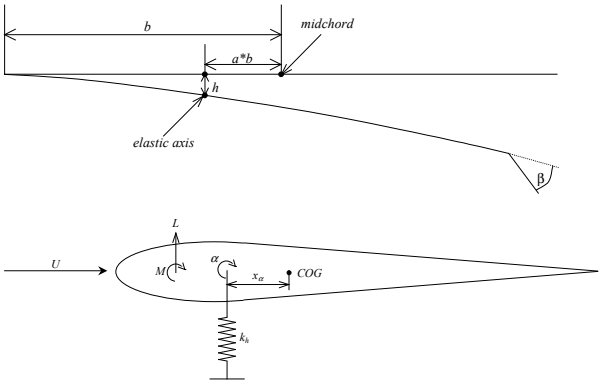

Fig. 1. The aeroelastic wing section

$$\mathbf{u}=-K\overset{N}{\underset{n=1}{\otimes}}\mathbf{w}_n(p_n)\mathbf{x}. \tag{6}$$

K symbolizes the PDC design technique applied.

IV. Equations of Motion of an Aeroelastic Wing Section

In the followings, the above controller design theorem is applied to the flutter suppression of the prototypical aeroelastic wing section shown in Fig. 1. The airfoil is constrained to have two degrees of freedom, the plunge h and pitch α. The equations of motion of the system have been derived in many references (for example, see [16]) and can be written as

$$\begin{pmatrix}m & mx_\alpha b\\ mx_\alpha b & I_\alpha\end{pmatrix}\begin{pmatrix}\ddot{h}\\ \ddot{\alpha}\end{pmatrix}+\begin{pmatrix}c_h & 0\\ 0 & c_\alpha\end{pmatrix}\begin{pmatrix}\dot{h}\\ \dot{\alpha}\end{pmatrix}+\begin{pmatrix}k_h & 0\\ 0 & k_\alpha\end{pmatrix}\begin{pmatrix}h\\ \alpha\end{pmatrix}=\begin{pmatrix}-L\\ M\end{pmatrix} \tag{7}$$

where x_α is the non-dimensional distance between elastic axis and the center of mass; m is the mass of the wing; I_α is the mass moment of inertia; b is semi-chord of the wing, and c_α and c_h respectively are the pitch and plunge structural damping coefficients, and k_h is the plunge structural spring constant. Traditionally, there have been many ways to represent the aerodynamic force L and moment M, including steady, quasi-steady, unsteady and non-linear aerodynamic models. In this paper we assume the quasi-steady aerodynamic force and moment, see work [17]:

$$L=\rho U^2bc_{l_\alpha}\left(\alpha+\frac{\dot{h}}{U}+\left(\frac{1}{2}-\alpha\right)b\frac{\dot{\alpha}}{U}\right)+\rho U^2bc_{l_\beta}\beta \tag{8}$$

$$M=\rho U^2b^2c_{m_\alpha}\left(\alpha+\frac{\dot{h}}{U}+\left(\frac{1}{2}-\alpha\right)b\frac{\dot{\alpha}}{U}\right)+\rho U^2bc_{m_\beta}\beta.$$

The above L and M are accurate for the class of low velocity. Wind tunnel experiments are carried out in [1]. In the above equation ρ is the air density, U is the free stream velocity, $c_{l\alpha}$ and $c_{m\alpha}$ respectively are the lift and moment coefficients per angle of attack, and $c_{l\beta}$ and $c_{m\beta}$, respectively are lift and moment coefficients per control surface deflection, and a is non-dimensional distance from the midchord to the elastic axis. Several classes of non-linear stiffness contributions $k_\alpha(\alpha)$ have been studied in papers treating the open-loop dynamics of aeroelastic systems

[17]. For purposes of illustration, we now introduce the use of polynomial non-linearities:

$$k_\alpha(\alpha) = k_{\alpha_0} + \sum_{i=1}^{I} k_{\alpha_i}\alpha^i .$$

The equations of motion derived above exhibit limit cycle oscillation, as well as other non-linear response regimes including chaotic response [17]. The system parameters to be used in this paper are tabulated in Table 1. These data are obtained from experimental models described in full detail in works by [1,17]. The non-linear stiffness term $k_\alpha(\alpha)$ is obtained by curve fitting the measured displacement-moment data for non-linear spring as [17]:

$$k_\alpha(\alpha) = 2.82(1 - 22.1\alpha + 1315.5\alpha^2 + 8580\alpha^3 + 17289.7\alpha^4.$$

Combining equations (1) and (2) one obtains

$$\begin{pmatrix} m & mx_\alpha b \\ mx_\alpha b & I_\alpha \end{pmatrix}\begin{pmatrix} \ddot{h} \\ \ddot{\alpha} \end{pmatrix} + \begin{pmatrix} c_h + \rho U b c_{l_\alpha} & \rho U b^2 c_{l_\alpha}\left(\frac{1}{2} - a\right) \\ \rho U b^2 c_{m_\alpha} & c_\alpha - \rho U b^3 c_{m_\alpha}\left(\frac{1}{2} - a\right) \end{pmatrix}\begin{pmatrix} \dot{h} \\ \dot{\alpha} \end{pmatrix} +$$
$$\begin{pmatrix} k_h & \rho U b^2 c_{l_\alpha} \\ 0 & -\rho U^2 b^2 c_{m_\alpha} + k_\alpha(\alpha) \end{pmatrix}\begin{pmatrix} h \\ \alpha \end{pmatrix} = \begin{pmatrix} \rho b c_{l_\beta} \\ \rho b^2 c_{m_\beta} \end{pmatrix} U^2 \beta \tag{9}$$

For the controller design, it is useful to convert the above equation into state-space formulation. Let

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} h \\ \alpha \\ \dot{h} \\ \dot{\alpha} \end{pmatrix} \text{ and } \mathbf{u} = \beta$$

Then we have

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{p})\mathbf{x} + \mathbf{B}(\mathbf{p})\mathbf{u} = \mathbf{S}(\mathbf{p})\begin{pmatrix} \mathbf{x} \\ \mathbf{u} \end{pmatrix}, \tag{10}$$

where

$$\mathbf{A}(\mathbf{p}) = \begin{pmatrix} x_3 \\ x_4 \\ -k_1 x_1 - \left(k_2 U^2 + p(x_2)\right)x_2 - c_1 x_3 - c_2 x_4 \\ -k_3 x_1 - \left(k_4 U^2 + q(x_2)\right)x_2 - c_3 x_3 - c_4 x_4 \end{pmatrix} \quad \mathbf{B}(\mathbf{p}) = \begin{pmatrix} 0 \\ 0 \\ g_3 U^2 \\ g_4 U^2 \end{pmatrix}$$

and $\mathbf{p}$ contains values x_2 and U. The new variables are tabulated in Table 2.

V. Quadratic Stability of the Aeroelastic Wing Section

The controller design of the aeroelastic system is performed according to the methodology discussed in Section III. First of all the transformation space Ω is defined. We are interested in the interval $U \in [14, 25]$(*m*/*s*) and we presume that the interval $\alpha \in [-0.03, 0.03]$(*rad*) is sufficiently large enough (note that these intervals can arbitrarily be set). Therefore let

Ω: [-0.03, 0.03] x [14, 25]

in the present example. Executing the TP transformation shows that dynamic model (10) can be represented exactly in multiple-model form (4) over a 2x3 "minimal" basis. TP transformation also shows that the definition of the convex basis needs to introduce an additional basis function on the second dimension. This means that the model in (10) can be described exactly by the convex combination of 2x4=8 linear vertex systems. The 3rd basis function of the "minimal" basis system on the second dimension has negligible contribution. We can simply discard the 3rd basis function at the risk of a small transformation error. This will tend to simplify the localization of the basis. After discarding one basis function the convex form needs again an additional basis function. The TP transformation then yields 2x3=6 vertex systems and "close-to-localized" convex basis.

Having the multiple-model form we can execute the PDC design techniques. Let us select one of the simplest PDC techniques that does not consider any constraint on the speed of the controller, the state vector and the control values, and does not involve LMI's: First we execute pool-replacement technique to define the vertex feedback systems, then we utilize Theorem C14 (quadratic stability) in the paper written by [18] to check the stability of the controlled multiple model: Dynamic system (4) where (6) holds is quadratic stable if and only if the following condition holds:

$$\mathrm{Re}\,\lambda_i(\mathbf{H}) \neq 0, \tag{11}$$

$d = m\left(I_\alpha - mx_\alpha^2 b^2\right)$	
$k_1 = \frac{1}{d} I_\alpha k_h$	$k_2 = \frac{1}{d}\left\{I_\alpha \rho b c_{l_\alpha} + mx_\alpha \rho b^3 c_{m_\alpha}\right\}$
$k_3 = -\frac{1}{d} mx_\alpha b k_h$	$k_4 = -\frac{1}{d}\left\{mx_\alpha \rho b^2 c_{l_\alpha} + m\rho b^2 c_{m_\alpha}\right\}$
$p(x) = -\frac{1}{d} mx_\alpha b k_\alpha(x)$	$q(x) = \frac{m}{d} k_\alpha(x)$
$c_1 = \frac{1}{d}\left\{I_\alpha\left(c_h + \rho U b c_{l_\alpha}\right) + mx_\alpha \rho U b^3 c_{m_\alpha}\right\}$	$c_2 = \frac{1}{d}\left\{I_\alpha \rho U b^2 c_{l_\alpha}\left(\frac{1}{2} - \alpha\right) - mx_\alpha b c_{l_\alpha} + mx_\alpha \rho U b^4 c_{m_\alpha}\left(\frac{1}{2} - \alpha\right)\right\}$
$c_3 = \frac{1}{d}\left\{I_\alpha\left(c_h + \rho U b c_{l_\alpha}\right) + mx_\alpha \rho U b^3 c_{m_\alpha}\right\}$	$c_4 = \frac{1}{d}\left\{mc_\alpha - mx_\alpha \rho U b^3 c_{l_\alpha}\left(\frac{1}{2} - \alpha\right) - m\rho U b^3 c_{m_\alpha}\left(\frac{1}{2} - \alpha\right)\right\}$
$g_3 = -\frac{1}{d}\left(I_\alpha \rho b c_{l_\alpha} - mx_\alpha b^3 \rho c_{m_\beta}\right)$	$g_4 = \frac{1}{d}\left(mx_\alpha b^2 \rho c_{l_\beta} + mb^2 \rho c_{m_\beta}\right)$

Table 1. System Variables

b	0.135m	ρ	1.225 kg/m^3
span	0.6m	c_{l_α}	6.28
k_h	2844.4 N/m	c_{m_α}	3.358
c_h	27.43 N/m	c_{l_β}	$(0.5+a)\,c_{l_\alpha}$
		c_{m_β}	-0.635

Table 2. System Parameters

where **H** is an indicator matrix. Its elements are detailed in [18].

VI. Result of the Anytime Control

To demonstrate the performance of the controlled system, numerical experiments are presented in this section. The numerical examples are performed with free stream velocity U=15 m/s. The controller is operated in an anytime manner which symbolizes the fluctuating resource/time availability. During 25% of the time only the most significant 3 vertex systems has been evaluated, in 25% 4, in 25% 5, while in the remaining 25% the total multiple model composed of the 6 vertex systems. In Figs. 2-3 the control signal can be followed if the number of the evaluated vertex systems is minimum 3, which can be compared to the results in Figs. 4-5 showing the control signal if the total system, consisting of 6 vertex systems is operated. Similarly, the time responses of the pitch and plunge are depicted in Figs. 6-7 and 8-9, and Figs. 10-11 and 12-13, respectively.

VII. Conclusions

In this paper we have extended a numerical control design methodology based on the TP model transformation and PDC design methods towards anytime use and applied it to design non-linear controllers for a prototypical aeroelastic wing section, which includes structural nonlinearities, such as chaotic behavior and limit cycle oscillation. The designed controller is able to stabilize quadratically the system using only one control surface even if only a significantly truncated model is evaluated instead of the original "exact" controller model.

VIII. Acknowledgment

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IX. References

[1] Block, J.J., T.W. Strganac, "Applied active control for nonlinear aeroelastic structure," *Journal of Guidance, Control, and Dynamics,* 1996.

[2] Ko, J., A.J. Kridula, T.W. Strganac, "Nonlinear Control Theory for a Class of Structural Nonlinearities in a Prototypical Wing Section," *AIAA Papers 97-0580*, pp. 1181-1189, 1997.

[3] O'Neil, T., H.C. Gilliat, T.W. Strganac, "Investigations of aeroelastic response for a system with continuous structural nonlinearities," *AIAA Papers 96-1390*, 1996.

[4] Ko, J., A.J. Kridula, T.W. Strganac, "Nonlinear dynamics and control for a structurally nonlinear aeroelastic system," *AIAA Papers 97-1024*, 1997.

[5] Yim, W.W., S.N. Singh, W.R. Wells, "Nonlinear Control of a Prototypical Aeroelastic Wing Section: State Dependant Riccati equation Method," *Int. Conference on Nonlinear Problems in Aviation and Aerospace,* FIT, Florida, 2002.

[6] Várkonyi-Kóczy, A.R., T. Kovácsházy, "Anytime Algorithms in Embedded Signal Processing Systems," *IX. European Signal Processing Conference, EUSIPCO-98*, Rhodes, Greece, Sep. 8-11, 1998, Vol. 1, pp. 169-172.

[7] Baron,C., J.-C Geffroy, G. Motet ed., *Embedded System Applications*. Kluwer Academic Publishers, 1997.

[8] Baranyi, P, "HOSVD based TP model transformation as a way to lmi based controller design," *IEEE Trans. on Industrial Electronics* (accepted).

[9] Tanaka, K., H.O. Wang, *Fuzzy Control Systems Design and Analysis*. John Wiley & Sons, Inc. New York, 2001.

[10] S. Zilberstein, "Using Anytime Algorithms in Intelligent Systems," *AI Magazine*, Vol. 17, No. 3), 1996, pp. 73-83.

[11] Várkonyi-Kóczy, A.R., T. Kovácsházy, O. Takács, Cs. Benedecsik, "Anytime Algorithms in Intelligent Measurement and Control," *2000 World Automation Congress*, WAC'2000, Maui, USA, June 11-16, 2000.

[12] Várkonyi-Kóczy, A.R., A. Ruano, P. Baranyi, O.Takács, "Anytime Information Processing Based on Fuzzy and Neural Network Models," *2001 IEEE Instrumentation and Measurement Technology Conference, IMTC/2001*, Budapest, Hungary, May 21-23, 2001, pp. 1247-1252.

[13] Boyd, S., et.al., "Linear Matrix Inequalities in System and Control Theory," *SIAM,* Philadelphia, 1994.

[14] L.D. Lathauwer, B.D. Moor, J. Vanderwalle, "A Multi Linear Singular Value Decomposition," *SIAM Journal on Matrix Analysis and Applications,* Vol 21, No. 4, pp. 1253-1278, 2000.

[15] D. Tikk, "On nowhere denseness of certain fuzzy controllers containing prerestricted number of rules," *Tatra Mountains Math. Publ.*, pp. 369-377, 1999.

[16] Y.C. Fung, *An Introduction to the Theory of Aeroelasticity,* John Wiley and Sons, New York, 1955.

[17] T. O'Neil, T.W. Strganac, "An Experimental Investigation of Nonlinear Aeroelastic Response," *AIAA Journal of Aircraft* (accepted).

[18] K. Tanaka, K., T. Ikeda, H.O. Wang, "Robust Stabilization of a Class of Uncertain Nonlinear Systems via Fuzzy Control: Quadratic Stabilizability, H_∞ Control Theory, and Matrix Inequalities," *IEEE Trans. on Fuzzy Systems*, Vol. 4, No. 1, 1996.

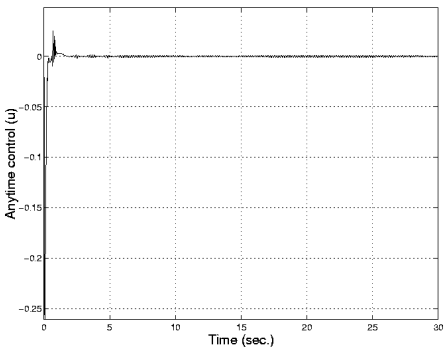

Fig. 2. Control signal using min. 3 vertex systems

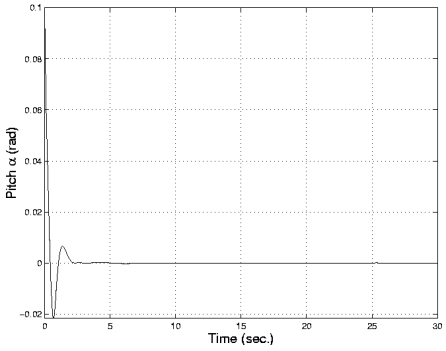

Fig. 6.Time response of the pitch using min. 3 vertex systems

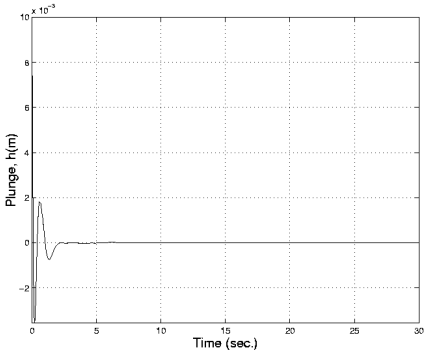

Fig. 10. Time response of the plunge using min. 3 vertex systems

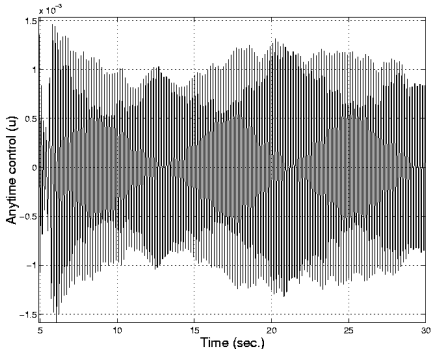

Fig. 3. Control signal using min. 3 vertex systems

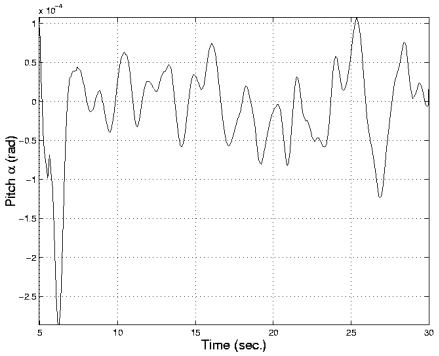

Fig. 7. Time response of the pitch using min. 3 vertex systems

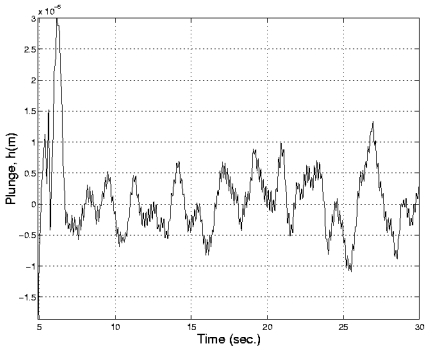

Fig. 11. Time response of the plunge using min. 3 vertex systems

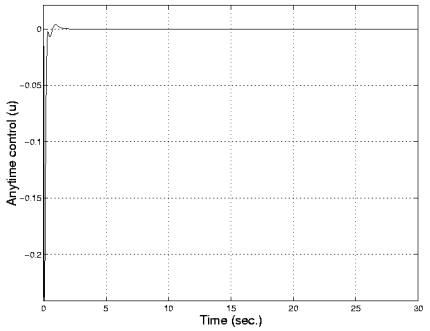

Fig. 4. Control signal using 6 vertex systems

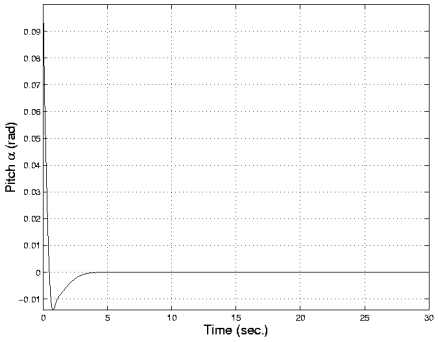

Fig. 8. Time response of the pitch using 6 vertex systems

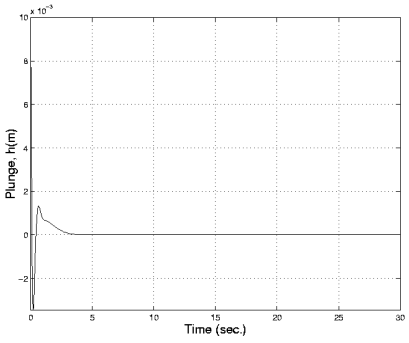

Fig. 12. Time response of the plunge using 6 vertex systems

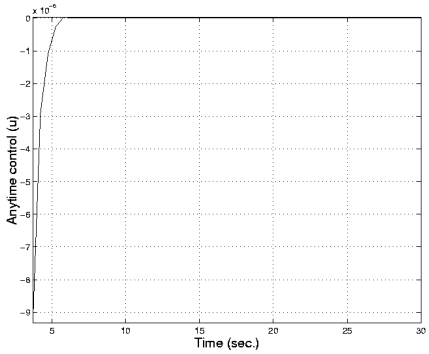

Fig. 5.Control signal using 6 vertex systems

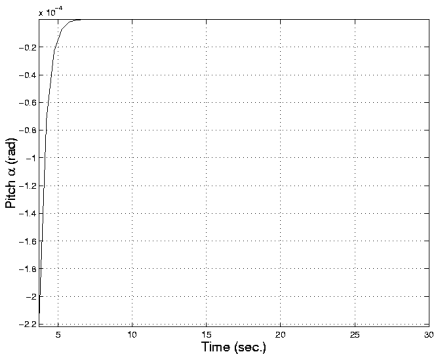

Fig. 9. Time response of the pitch using 6 vertex systems

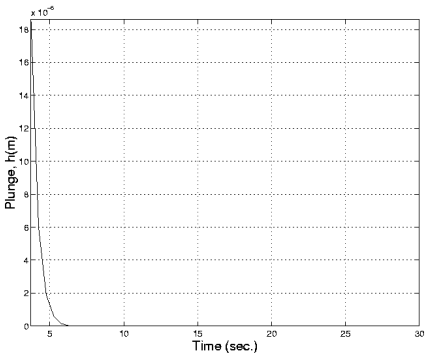

Fig. 13. Time response of the plunge using 6 vertex systems